8,

REMARKS

ON

MAJOR-GENERAL ROY'S ACCOUNT

OF THE

TRIGONOMETRICAL OPERATION.

By Mr. ISAAC DALBY.

From the PHILOSOPHICAL TRANSACTIONS.

ERRATA.

P. 229. laſt column of the Table, and laſt line but one of that column, *for* 17 36 *read* 17 16

P. 250. Triangle XIII. diſtance of Norwood from Chiſlehurſt Church in feet, *for* 35777.9 *read* 36777.9.

N. B. Page 111. of this Volume correſponds to page 3. of General Roy's ſeparate Copies.

B L

OUR late much reſpected Colleague, Major-General ROY, having finiſhed, in September 1788, the trigonometrical meaſurement deſcribed in the firſt Part of this Volume, returned to London in a very indifferent ſtate of health. From this time he employed all the leiſure that his illneſs, and his various official avocations, allowed, in preparing the account of his operations, to be laid before the Royal Society. But toward the autumn of 1789 his infirmities encreaſed ſo much, that the medical Gentlemen he conſulted adviſed him to ſpend the following winter at Liſbon, for which place he accordingly embarked in the beginning of November. Previous to this, however, he finiſhed the firſt Copy of his Paper; but it was much hurried toward the latter part, and not rendered ſo perfect as the General would undoubtedly have made it with more time and better health. He returned to England in April 1790, and the Paper was ſent to the preſs before the end of the ſame month. Unfortunately the General did not live to ſee the printing quite completed; he corrected, indeed, all the ſheets except the three laſt; but without comparing his manuſcript copy with the original papers and obſervations. Several errors which had been diſcovered in the courſe of the printing, together

ther with the obſcurity of the account in certain parts, induced ſome of the General's Friends, Members of the Royal Society, to requeſt, after his deceaſe, that the whole might be reviſed by a competent perſon, who ſhould compare it with the original documents, correct ſuch miſtakes as might be diſcovered, and illuſtrate whatever required further explanation. No one could be found ſo proper for this taſk as Mr. DALBY, the Gentleman of whom the General makes ſuch honourable mention in his Paper, and who, having aſſiſted in all the operations, was as well acquainted with every part of them as the General himſelf. The reſult of Mr. DALBY's examination is the following Remarks; which being much too long for inſertion in the liſt of *errata* (where only the errors of the preſs are noticed) is here added ſeparately, by way of Appendix.

C. BLAGDEN.

REMARKS, &c.

PAGE 134. l. 20, &c. The inclinations of the baſes with the meridians were determined by ſpherical computation, and therefore can only be conſidered as *nearly* true.

P. 171. l. 6. from bottom, *for* we have area in feet *put* we have log. area in feet.

P. 173. in the VIII triangle, *for* 0.01 *put* 0.1.

P. 174. in the IX triangle, *for* 0.88 *put* 0.83.

P. 175. in the XVII triangle, *for* 71855 *put* 71885.

P. 177. This method of making the compariſon on a long diſtance, when the meaſured baſes are ſhort, and nearly of the ſame length, ſeems preferable to that of carrying the computation directly from one baſe to the other. Determinations of this kind, however, muſt always be uncertain to particular limits on account of the inaccuracy of inſtruments and obſervations combined with the unknown figure and dimenſions of the earth. Was the earth a ſphere of a known magnitude, the moſt natural method of computation would have been by ſpherical trigonometry, after the obſerved angles had been corrected for that purpoſe; which method (ſuppoſing the angles wanted no correction, or each had been accurately obſerved) would ſhew which-baſe was meaſured neareſt the truth. Or the ſame thing might alſo be obtained by plane trigonometry,

try, uſing the chords of the meaſured baſes, and the angles formed by the chords of the other ſides of the triangles (which angles might then be found from the horizontal ones) inſtead of the obſerved angles.

In the application of plane trigonometry to the obſervations, that part of the earth's ſurface to which the operation has been confined, is conſidered as a plane, and the meaſured baſes as right lines on that plane; but whether the computations are made on this principle, or attempted on that of taking the baſes and the other ſides of the triangles as chords, there ſeems to be no certain rule for reducing the obſerved angles of each triangle to 180°, ſo as to give the diſtances the moſt correct, which would alſo be the caſe if the angles of each triangle had been taken in the ſame plane; hence it is evident, that the method of correction has been in ſome degree arbitrary; for, though the ſum of the three obſerved angles of each triangle is in general very near what it ought to be (taking the earth as a ſphere); yet, when that ſum is not exactly 180°, in reducing them to plane ones, each obſerved angle may be taken as a plane one, and the other two augmented in caſe of a defect; but each ought to be diminiſhed when there is an exceſs in the obſerved ſum. In making theſe reductions, however, it is evidently neceſſary to conſider whether each of the obſerved angles is equally well aſcertained, and correct them accordingly; but this muſt be left to the judgement of the obſerver.

From the foregoing conſiderations, it follows, that the angles of the triangles taken as plane ones may be varied to certain limits, and conſequently the oppoſite ſides deduced therefrom muſt vary to certain limits alſo; but it is evident, that a mean of the extreme reſults, obtained in this manner, will be very near the truth; and therefore this method of making the compariſon ſeems leſs liable to objection than that by a ſingle correction of the ſame angles. Accordingly, if we vary the angles (in reducing them to 180°) from Hounſlow Heath to the XIII triangle, ſo as to produce the greateſt and leaſt effects on the lengths of the oppoſite ſides, there will reſult 141750½ and 141746½ feet, nearly, for the greateſt and leaſt, and 141748½ for the mean diſtance of Fairlight and Hollingborn. In like manner, the baſe on Romney Marſh will give 141745.6 and 141744.4 feet, nearly, and the mean 141745 feet, for the diſtance of the ſame ſtations; the difference in the mean reſults is 3½ feet on a diſtance of near 27 miles; and therefore the baſe on Hounſlow Heath meaſures the other, by theſe determinations, to about 9 inches; and, becauſe the latter baſe is the longeſt, it would meaſure the former on Hounſlow Heath to ſomething leſs.

The diſtance of the ſtations of Fairlight and Hollingborn in the XIII triangle is 141747.1 feet, and from this all the diſtances to the eaſtward are computed; but if the baſes are meaſured equally exact, the diſtance of the above ſtations, or 141745 feet, determined from the baſe on Romney Marſh, muſt be more correct than the other, becauſe the connection of Fairlight and Hollingborn with this baſe is formed by three or four triangles only, whereas on the other ſide, the computation runs through eight or nine. The difference, however, is but 2 feet, and that in an extent of almoſt 27 miles, which will make about 7½ feet leſs for the diſtance between the meridians of Greenwich and Paris.

Among the angles corrected for computation, it will be perceived, that ſometimes the quantity of an angle ſeems not to be exactly what the obſerved angle ought to give. In theſe caſes the obſerved angle is leſs to be depended on than the others of the ſame triangle: for inſtance, in the firſt triangle the obſerved angle at Hanger Hill is 42° 2′ 32″, and that for computation 42° 2′ 34″; now this angle was not taken ſo accurately as could be wiſhed, but the others were repeatedly obſerved.

P. 178. in the XXIX triangle, *for* 186113 *put* 147386.9.

P. 180. in the XXXIV. triangle, *for* 252469.9 *put* 252496.9.

As the obſervations at Fairlight and Dover are omitted, upon which the angles of the XXXIII and XXXIV triangles depend, it will be neceſſary to give them, and alſo ſhew the manner of obtaining theſe angles.

	°	′	″
At Dover Caſtle, the angle between the white light at Montlambert and the lamp at Padleſworth was obſerved	109	8	25.5
Corrected for computation . . .	109	8	25
At Fairlight, the angle between the white lights at Montlambert and Blancnez was obſerved	17	46	5
For computation . . .	17	46	3.5
Between the lamp at Lydd and white light at Blancnez	18	2	31
For computation . . .	18	2	31.5

The acute angles in the XXXII triangle reſult from the other angle and including ſides 147386.9 and 42561.18.

		°	′	″
Angle at Fairlight	in the XXIX triangle	13	38	2.95
	in the XXXII triangle	6	6	39.43
Angle at Fairlight between Dover and Lydd		7	31	23.52
Angle at Fairlight between Lydd and Montlambert (17° 46′ 3″½ + 18° 2′ 31″½) .		35	48	35
Angle at Fairlight in the XXXIII triangle		43	19	58.52

	°	′	″
Angle at Dover between Padleſworth and Montlambert . . .	109	8	25
—— at Dover in the XXXII triangle	21	37	55.42
Angle at Dover in the XXXIII triangle .	87	30	29.58

The third angle, or 49° 9′ 31″.9 at Montlambert, is the ſupplemental one.

If from the angle at Fairlight in this triangle we take 17° 46′ 3″½ we have 25° 33′ 55″.02 for the angle at Fairlight in the XXXIV triangle; and if to 87° 30′ 29″.58 we add 23° 25′ 0″.25 (the angle at Dover in the XXXV triangle) it gives 110° 55′ 29″.83 for the angle at Dover; that at Blancnez is the ſupplemental one.

The ſituation of the ſtation at Montlambert, as determined by the obſervations made on this ſide of the Channel, has not however totally depended on thoſe made at Fairlight and Dover; another obſervation at Padleſworth has been uſed by way of check, or verification. This was made in a very favourable ſtate of the air, when the angle between the flag-ſtaff at Dover and maſt at Montlambert was found to be 58° 27′ 11″½, which is nearly what reſults from computation; for 42561.18 and 168821.07 feet, the reſpective diſtances of Dover from Padleſworth and Montlambert, with the included angle 109° 8′ 25″, give this angle 58° 27′ 10″.9.

It ought to be remarked, that the angle at Blancnez 119° 41′ 28″.9, communicated by M. Cassini, is an horizontal one; that of the XXXV triangle, or 119° 41′ 41″.6, is the reſult of a computation by plane trigonometry, which, if accurate, ſhould be leſs than the horizontal one at the ſame point, and therefore the *maximum* of the difference muſt be ſomewhat greater than 12″.7.

P.

P. 182. &c. The triangles after the XXXVI, and what follows to the end of the article, do not ſeem neceſſary, on our part, to complete the triangular connection between Greenwich and Paris, becauſe it may be done in the following manner from the triangle formed by Dover, Calais, and Dunkirk. In this triangle, the ſide between Dover and Calais is 137449.9 feet (ſee the XXXVI triangle); and by M. Cassini's Paper, communicated in 1789, it appears, that the ſide between Calais and Dunkirk is 19349.34 toiſes (= 123729.3 feet), and the included angle at Calais 139° 17′ 35″.6; theſe give 19° 14′ 13″.1 and 21° 28′ 11″.3 for the other two angles, and 244919 feet for the diſtance of Dover and Dunkirk; alſo 28232.7 for what Dunkirk is ſouthward, and 243287 feet for what it is eaſtward, from Dover: this laſt added to 307366.8, the diſtance of Dover from the meridian of Greenwich, gives 547053.8 feet for what Dunkirk is eaſt from the meridian of Greenwich on a parallel to the perpendicular; but the length of the arc of the great circle which paſſes through Dunkirk, and is perpendicular to the meridian of Greenwich, is very near the ſame as the length of this parallel, or 547053.8 feet (though accurately ſomewhat leſs); hence, if we take 61247½ fath. = 1°, we have 1° 29′ 19″.1 for that arc, and the latitude of Dunkirk being 51° 2′ 9″.3 (See Sect. VI. Art. 12.); therefore, coſine 51° 2′ 9″.3 : rad. :: ſine 1° 29′ 19″.1 : ſine 2° 22′ 3″.8 the longitude of Dunkirk (agreeing with that in Art. 12.); and rad. : tang. 51° 2′ 9″.3 :: tang. 1° 29′ 19″.1 : coſine 88° 9′ 34″ for the other angle of this triangle of which the co-latitude of Dunkirk is the hypothenuſe.

Dunkirk is eaſt from the meridian of Paris 1416, or 1430 toiſes (ſee Art. 12. Sect. VI.), a mean of theſe give 1′ 29″.14 as an arc of a great circle; this, with the co-latitude of Dunkirk for the hypothenuſe, will give 89° 58′ 10″ for the angle between the meridian and this arc; hence 89° 58′ 10″ − 88° 9′ 34″ = 1° 48′ 36″ is the angle at Dunkirk between the two arcs; one perpendicular to the meridian of Greenwich, and the other to that of Paris; therefore, if we take 1416, or 1430 toiſes, as the leg of a plane triangle adjacent to this angle, we get 9059, or 9148½ feet, for the diſtance of the meridian of Paris from Dunkirk on a great circle perpendicular to the meridian of Greenwich; theſe taken from 547053.8 will leave 537994.8 or 537905.3 feet, for the diſtance between the meridians of Greenwich and Paris on that circle, according as Dunkirk is 1416 or 1430 toiſes from the meridian of Paris.

Should it be thought more accurate to make uſe of the diſtance between Calais and Dunkirk, according to the ſcale in the XXXV triangle deduced from the Engliſh obſervations acroſs the Channel, it is had at one proportion thus; as 12077.85 toiſes (the French diſtance between Blancnez and Montlambert) is to 77235 feet (the Engliſh diſtance) ſo is 19349.34 toiſes (the French diſtance of Calais and Dunkirk) to 123734.5 feet, as in the XL triangle, which exceeds the French diſtance about 5 feet: this will give the diſtance between the meridians of Grenwich and Paris 4.4 feet more than the above determination. But was the baſe on Romney Marſh adhered to, it would give the diſtance about 3 feet leſs; and therefore the reſults of the French triangles on their coaſt would agree nearer with the deductions from this baſe than from the other on Hounſlow Heath.

P. 187. in lines 1. and 8. *for* 358.6 *put* 349.4; and conſequently in line 10. *for* 133409.8 *put* 133419. This error is the cauſe of the difference in the diſtances of the parallels of latitude of Greenwich and Paris as given in Art. 10. and 13. Sect. VI.:

for, in pages 185. 187. we have 133746.3 and 133768.4 fath. the diſtances of Dunkirk north from Paris, the mean is 133757.4; if from this we take 358.6, we have 133398.8, as in Art. 10.; but ſubtracting 349.4, gives 133408 for what M is north of Paris, which added to 27248.2 there reſults 160656.2 agreeing with that in Art. 13.

P. 190, 191. It will immediately be perceived, that the columns in the table of general reſults here alluded to, have been filled by a method ſimilar to that of working a *traverſe*. The following table, however, was previouſly drawn up to facilitate the computations, and by which the numbers in the two firſt columns of diſtances may at any time be eaſily examined.

This table will readily be underſtood; for, if we ſuppoſe parallels to the meridian of Greenwich to be drawn through the ſtations on the left, oppoſite on the right are the angles which the adjacent ſtations make with theſe parallels.

		°	′	″	
Greenwich Obſ.	Norwood .	38	7	16	SW
Norwood .	Greenwich Obſ.	38	7	16	NE
	Hundred Acres	42	22	39.2	SW
	Hanger Hill .	49	31	22.7	NW
Hundred Acres .	Norwood .	42	22	39.2	NE
	Hanger Hill .	19	50	1.7	NW
	St. Ann's Hill	73	48	38.3	NW
Hanger Hill :	Norwood .	49	31	22.7	SE
	Hundred Acres	19	50	1.7	SE
	Hampton Poor-houſe	23	30	53.4	SE
	St. Ann's Hill .	48	34	42.2	SW
	King's Arbour :	65	33	27.4	SW

Hampton

		°	′	″	
Hampton Poor-houſe	Hanger Hill .	23	30	53.4	NE
	King's Arbour .	44	24	45.6	NW
	St. Ann's Hill .	74	8	40.9	SW
King's Arbour .	Hanger Hill .	65	33	27.4	NE
	Hampton Poor-houſe	44	24	45.6	SE
	St. Ann's Hill .	29	49	49.4	NW
	Windſor .	87	29	43.1	NW
St. Ann's Hill .	King's Arbour .	29	49	49.4	NE
	Hanger Hill .	48	34	42.2	NE
	Hampton Poor-houſe	74	8	40.9	NE
	Hundred Acres .	73	48	38.3	SE
	Windſor .	29	19	24.6	NW
Greenwich Obſ. .	Severndroog Caſtle	73	49	34	SE
Severndroog Caſtle	Greenwich Obſ.	73	49	34	NW
	Botley Hill .	11	23	18.5	SW
	Wrotham Hill .	46	18	30	SE
Botley Hill .	Severndroog Caſtle	11	23	18.5	NE
	Wrotham Hill .	79	16	28.7	NE
	Goudhurſt .	60	38	49.3	SE
	Frant .	43	28	20.3	SE
Frant .	Botley Hill .	43	28	20.3	NW
	Wrotham Hill .	6	50	57.7	NE
	Hollingborn Hill	55	19	35.3	NE
	Goudhurſt .	82	24	11.4	NE
	Fairlight Down	45	17	22.7	SE
Wrotham Hill .	Severndroog Caſtle	46	18	30	NW
	Botley Hill .	79	16	28.7	SW
	Frant .	6	50	57.7	SW
	Goudhurſt .	25	12	14.7	SE
	Hollingborn Hill	77	21	25.7	SE

		°	′	″	
Goudhurſt .	Hollingborn Hill	38	51	47.3	NE
	Tenterden .	72	54	53.3	SE
	Fairlight Down .	23	15	17.5	SE
	Frant .	82	24	11.4	SW
	Botley Hill .	60	38	49.3	NW
	Wrotham Hill .	25	12	14.7	NW
Fairlight Down .	Frant .	45	17	22.7	NW
	Goudhurſt .	23	15	17.5	NW
	Hollingborn Hill	3	8	32.3	NE
	Tenterden .	12	5	40.9	NE
	Allington Knoll	45	46	21.3	NE
	Lydd .	67	4	58.3	NE
	Blancnez .	85	7	29.7	NE
	Montlambert .	77	6	26.7	SE
Hollingborn Hill	Wrotham Hill	77	21	25.7	NW
	Frant .	55	19	35.3	SW
	Goudhurſt .	38	51	47.3	SW
	Fairlight Down .	3	8	32.3	SW
	Tenterden .	5	46	56.8	SE
	Allington Knoll	45	47	55.7	SE
Tenterden Church	Hollingborn Hill	5	46	56.8	NW
	Goudhurſt .	72	54	53.3	NW
	Fairlight Down	12	5	40.9	SW
	Lydd .	50	27	11.6	SE
	Allington Knoll	85	47	25.3	NE
Lydd Church .	Fairlight Down	67	4	58.3	SW
	Tenterden .	50	27	11.6	NW
	Ruckinge :	6	16	20.4	NW
	Allington Knoll	12	46	58.3	NE
	High Nook .	37	4	28.1	NE
	Padleſworth .	41	10	52.6	NE
	Folkſtone Turnpike	50	49	22	NE

Ruckinge

		°	′	″	
Ruckinge	Lydd	6	16	20.4	SE
	High Nook	55	15	9.8	SE
	Allington Knoll	70	25	31.6	NE
Allington Knoll	Hollingborn Hill	45	47	55.7	NW
	Tenterden	85	47	25.3	SW
	Fairlight Down	45	46	21.3	SW
	Lydd	12	46	58.3	SW
	High Nook	21	1	47.8	SE
	Folkſtone Turnpike	82	56	18.9	NE
High Nook	Allington Knoll	21	1	47.8	NW
	Ruckinge	55	15	9.8	NW
	Lydd	37	4	28.1	SW
	Folkſtone Turnpike	58	39	12.6	NE
	Padleſworth	43	50	47.1	NE
Padleſworth	High Nook	43	50	47.1	SW
	Lydd	41	10	52.6	SW
	Folkſtone Turnpike	64	18	47.4	SE
	Dover Caſtle	81	11	30.1	NE
	Swingfield	44	47	7.1	NE
Folkſtone Turnpike	Lydd	50	49	22	SW
	High Nook	58	39	12.6	SW
	Allington Knoll	82	56	18.9	SW
	Padleſworth	64	18	47.4	NE
	Swingfield	3	51	7.8	NW
	Dover Caſtle	65	52	45.6	NE
Swingfield	Padleſworth	44	47	7.2	SW
	Folkſtone Turnpike	3	51	7.8	SE
	Dover Caſtle	79	27	47.8	SE

		°	′	″	
Dover Caſtle	Swingfield	79	27	47.8	NW
	Padleſworth	81	11	30.1	SW
	Folkſtone Turnpike	65	52	45.6	SW
	Montlambert	27	56	54.8	SE
	Blancnez	51	21	55.1	SE
	Calais	64	8	37.1	SE
	Dunkirk	83	22	52.9	SE
	——— by M. CASSINI's diſtance of Dunkirk and Calais	83	22	50.2	
	Point M	82	33	14.4	SE

The difference between the complement of 82° 33′ 14″.4 and the angle at M in the XL triangle is 14° 51′ 3″.9, the angle RMC referred to in Art. 10. and 11. Sect. VI.

P. 194. It is ſaid, that the angle AB*v* (Pl. X. fig. 2.) is equal to the angle BA*r*, and conſequently at p. 199. that the ſum of the obſerved angles PAB, PBA, are equal to the ſum that would be found on a ſphere. This (though extremely near in any of the ſpheroids hitherto aſſumed for the figure of the earth) is not geometrically true when the points of obſervation are *on* the ſurface of the ſpheroid, and each angle taken exactly in the plane of the horizon. For, it is evident, that to have the ſum *accurately* the ſame, the points A, B (the places of obſervation) muſt be at equal diſtances from G and W; and therefore, if at any two points thus taken in the verticals GA, WB, the angles are ſuppoſed to be in planes parallel to the reſpective horizons at A and B, their ſum will always be the ſame. Hence, becauſe the vertical WB is greater than GA, if the angles are accurately horizontal ones at A and B, their ſum muſt be *greater* on the north ſide, and *leſs* on the ſouth, than on a ſphere, except the latitudes of A and B are the ſame. The difference, however, is ſo minute, that for practical purpoſes they may be conſidered as equal (as in this Section and the corollary, p. 215.), without ſenſible error. In the example at p. 196. the difference in the ſum of the horizontal angles at A and B on this ſpheroid, and on a ſphere, is a ſmall fraction of a ſecond; but it requires a nice computation to diſcover the exact quantity. The method, however, is to compute the angle at B in the ſame manner as that is done at A; or by taking the point of obſervation in the vertical GA produced, 168 fathoms (the difference of the verticals WB, GA) above the ſurface at A, and determining the diminution in the horizontal angle by a re-computation.

By purſuing a method of computation ſimilar to that for the point A at p. 196. it is evident, that the three horizontal angles of any triangle on a known ſpheroid may be determined.

P. 195. bottom line, *for* AGH *put* AGK.

P. 203. There ſeems a miſtake towards the latter part of this page; becauſe it will be ſeen, that no ſuch ſpherical triangles have been uſed in the computations but in Art. III. p. 206.

P. 205. l. 13. from bottom. This muſt allude to one place of obſervation only; becauſe in this operation (where the latitudes have not been obſerved) a principal advantage lies in having one of the ſtations (like Botley Hill) on, or near the meridian of Greenwich, on account of obtaining its latitude pretty exact; but the farther off the other place of obſervation is, the better it is for the purpoſe.

P. 207. In l. 14. from bottom, *for* :: P✱ *put* :: ſine P✱.

P. 208. from l. 5. to the period in l. 12. from bottom, ſhould run thus:

If the latitude of the point B was given, and the earth a ſphere, the co-latitude BP and the obſerved angles PBG = 119° 21′ 13″.2, and PGB = 60° 17′ 15″.7, would give PG the co-latitude of G, and the angle BPG the difference of longitude of B and G.

Taking a ſphere whoſe diameter is nearly a mean between thoſe in M. BOUGUER's ſpheroid, the length of a degree of a great circle is 60859.1 fathoms, and the latitude of B will be 51° 16′ 41″.45; therefore BP = 38° 43′ 18″.54; this, with the obſerved angles at B and G, give PG = 38° 53′ 6″.72, and the angle BPG, or difference of longitude = 27′ 36″.7; therefore in the right-angled ſpherical triangle PRG, rad. : tang. GP :: coſine angle RPG : tang. 38° 53′ 3″.47 = RP; and rad. : ſine GP :: ſine RPG : ſine 17′ 20″ = RG.

P. 209. l. 9. *for* 51° 16′ 46″ *put* 51° 16′ 46″.1.

P. 213. Correct the title of this Article, by reading *geodetical meaſurement* for "*pole-ſtar obſervations*," in the firſt line.

P. 217. After the word "meridian", in the third line of Art. X. inſtead of "and alſo the differences of latitude and longitude have been obtained by very accurate obſervations of the pole-ſtar made at certain ſtations to the eaſtward of Greenwich,", *read* and alſo the difference of longitude between Botley Hill and Goudhurſt have been obtained by obſervations of the pole-ſtar.—A correction of this kind ſeems neceſſary, becauſe the pole-ſtar obſervations have not been uſed in finding the differences of latitude. From the directions of the meridians at the above ſtations, the value (in parts of a degree) of the meaſured arc of a great circle, perpendicular to the meridian, has been determined; hence the lengths of the degrees in the Table, p. 227. have been inferred. The diſtances from the meridian of Greenwich (in the Table of General Reſults) have been converted into degrees, &c. according to this Table; and the others from the perpendicular in the next column, according to M. BOUGUER's ſcale on the meridian (which is had ſufficiently accurate from the Table, p. 298. *Fig. de la T.* or that at p. 228. Phil. Tranſ. 1787, by an eaſy approximation) theſe meridional arcs applied to the co-latitude of Greenwich, with the other arcs perpendicular to the meridian, form the legs of the triangles by which the latitudes and longitudes of the ſtations have been computed. The meridional arcs, however, have been corrected, as in the example in this Article (where the value of R*r* has been added) when the diſtances of the ſtations from the meridian of Greenwich are conſiderable.

In determining the latitude of M in this Article, a ſpheroidical correction has been applied to the reſult by ſpherical trigonometry, as in Art. VII. but that computation is made on a figure of known dimenſions, and conſequently the latitude of *r* (fig. 7.) is given; but it does not follow, that the true latitude of *r* (fig. 10.) would exactly correſpond with M. BOUGUER's hypotheſis, though the length of the whole meridional arc between Greenwich and Paris is found to agree extremely near; and therefore no correction of this kind is applied to the other latitudes in the Table of General Reſults.

The greateſt accuracy, however, is abſolutely neceſſary in determining the directions of the meridians if we would derive ſatisfactory concluſions therefrom, when the places of obſervation are obliquely ſituated with reſpect to the meridian, and at a diſtance from each other not greater than that between Botley Hill and Goudhurſt, becauſe an error of 1″ in the horizontal angle at either of theſe places will produce an error in their difference of longitude of 1″.2 of a degree, and conſequently a variation of about 6″ in the longitude of Dunkirk or Paris.

Was the diſtance of the ſtations about 36 miles, the error in longitude would be the ſame as that in the horizontal angle, or 1″.

The length of the arc RG (Pl. X. fig. 5.) is 17695 fathoms, and its value 17′ 20″.06 as an arc of a great circle perpendicular to the meridian. Now, was the earth a ſphere, the length of any arc, would be to the number of degrees it contained, as 17695 to 17′ 20″.06; but this is not accurately the caſe on a ſpheroid; though, on this account only, the error in longitude (which is in defect) deduced from an arc of a great circle obtained in the above manner, muſt be ſmall to the extent of 3 or 4 degrees (in the latitudes of the places of obſervation) on a ſpheroid not more oblate than the earth.

It may be obſerved, that in determining the differences of longitude by the pole-ſtar obſervations, the ſtations ſhould be as nearly eaſt and weſt from each other as the nature of the country will permit, becauſe in that direction, any errors which may be thought to ariſe from the uncertain inclination of the verticals on the ſpheroid, will vaniſh; and, what is of more conſequence, a longer arc of a great circle perpendicular to the meridian will thereby be determined than could be in any other direction with the ſame diſtance. On this account the ſtations at Botley Hill and Hollingborn Hill (for one is ſeen from the other) are eligible. Their diſtance is about 28½ miles, which would meaſure near 24′ of a degree of a great circle perpendicular to the meridian.

P. 220. Art. XI. ſeems to want correction: for, if M*g* is a leſſer circle parallel to the meridian GR, it will cut the great circles *r*M, G*g*, at right angles. Hence, RMC − RM*r*

(14°

(14° 51′ 3″.9 − 19″.42) = 14° 50′ 44″.5 = *r*MC, which added to 90° (*r*M*g*) gives 104° 50′ 44″.5 for the angle *g*MC; from this take 1° 48′ 38″.6 (PM*g*), and we have 103° 2′ 5″.9 for the angle PMC deduced according to this method. But it cannot be ſaid to reſult from the Britiſh obſervations, becauſe the French angles were made uſe of to the eaſtward of Dover for obtaining the angle RMC, on which it depends.

P. 227. The lengths of the degrees of longitude in the Table were found thus: as radius : coſine of the latitude :: length of a degree of a great circle perpendicular to the meridian : length of a degree of longitude. This proportion is true on a ſphere, but not accurately ſo on a ſpheroid.

P. 229. *for* 43° 39′ *put* 48° 39′, the latitude of St. Malo.

As the new longitudes in this Table have not all been obtained in the ſame manner, it may not be improper to give the methods of computation.

The latitude of Straſbourg (*Deſcrip. Geom.*) is 48° 34′ 50″, and its diſtance from the meridian of Paris 204779 toiſes (=218243.17 fathoms) which, if we take 61225 fathoms = 1° (ſee the Table, p. 227.) gives 3° 33′ 52″.6; hence, as coſine 48° 34′ 50″ : rad. :: ſine 3° 33′ 52″.6 : ſine 5° 23′ 33″ the longitude.

In the *Connoiſſance des Temps* 1788, the latitude of Straſbourg is 48° 34′ 35″, longitude 5° 26′ 18″; therefore, as rad. : coſine 48° 34′ 35″ :: ſine 5° 26′ 18″ : ſine 3° 35′ 42″.3, the arc of the great circle (from which its longitude was computed) paſſing through Straſbourg, and falling perpendicular on the meridian of Paris.

According to the Advertiſement in the Map of France, the French computations have been made with a degree containing 57060 toiſes (=60811.7 fathoms); therefore, if we reduce

3° 35′ 42″.3 in the proportion of 61225 to 60811.7, we have 3° 34′ 14″.9 for the value of that arc when 61225 fathoms is = 1°; hence coſine 48° 34′ 35″ : rad. :: ſine 3° 34′ 14″.9 : ſine 5° 24′ 6″, the other longitude of Straſbourg. By the latter method the longitude of Cordouan was computed; but the other longitudes according to the former. The diſtances from the meridian of Paris are to be found in the publications alluded to in the above page.

P. 232. In the Table of General Reſults, *for* 1° 8′ 9″ and 4 m. 32 s. 36th. *put* 1° 8′ 4″ and 4 m. 32 s. 16th. the longitude of Padleſworth.

Againſt Calais, *for* 7 m. 23 s. 15.8th. *put* 7 m. 23 s. 15.2th.

Againſt Fairlight Down, in the laſt column but two, *for* 539.5 *put* 593.5.

P. 239. l. 15. *for* fig. 12. *put* fig. 13.

P. 242. l. 10. from bottom, *for* 3′ 55″ *put* fig. 3′ 35″.

P. 243. l. 13. *for* OKT *put* OK*t*. l. 5. from bottom, *for* *kt* put *kL*.

P. 224. l. 11. *for* 7″¼ *put* 7¼.

In the Table facing p. 246. in the column of *mean refraction*, for 0′ 15″.4 *put* 1′ 28″.1.

In addition to the examples of refraction in Sect. VII. the following (which was overlooked when this part was drawn up) may not improperly be given, as being of a different kind. It ſhews, that terreſtrial refraction (though often much *greater*) muſt, at particular times, be much *leſs* than is generally ſuppoſed.

Oct. 7, 1787, at the ſtation near Padleſworth, the depreſſion of the horizon of the ſea, in a SW direction nearly, was obſerved 26′ 27″. A degree of a great circle in this direction is about 61000 fathoms, and therefore 61000 × 6 × 57.2957795

=

= 20970255 feet, will be the radius of curvature nearly. The height of the ſtation above low-water ſpring-tides (as determined by alternate obſervations at this place and Dover Caſtle) was 642 feet; hence $\frac{20970255}{20970255+642} = ,9999693861$ the natural coſine of 26′ 54″ the *dip*; therefore 26′ 54″ − 26′ 27″ = 27″ is what the horizon was elevated by refraction. The ſtate of the tide however, is not taken into conſideration; but the time was about noon.

The weather was calm and cloudy, and the horizon clear. Barometer 29.6. Thermometer 70°, at 1 P.M.

The above obſervation was made with great care and attention.

P. 249. in the 3. triangle, *for* 76807.5 *put* 76812.4.

The operations alluded to in the note at the botttom of this page were,

		°	′	″
Angles obſerved at Spring-Grove Houſe between	Richmond Ob. and the Pagoda	13	10	13
	The Pagoda and St. Paul's	23	17	15
	St Paul's and Harrow Spire	74	18	43

Angles taken on Fulham Church between	Stretham Church and St. Paul's	70	24	52
	St. Paul's and Hampſtead Church	44	50	46
	Hamp. Ch. and Hanger H. Tower	57	27	43

In 1787, at Hanger Hill, the angle between Richmond Obſervatory and the Pagoda was obſerved 12° 26′ 42″. At the Hundred Acres, that between Hanger Hill and Batterſea Church 23° 59′ 44″; and that between Hanger Hill and Stretham Church 42° 3′ 57″. The angle at St. Paul's between Batterſea Church and the SW Pinnacle of Weſtminſter Abbey is 9′ 45″.6.

The reſults from the obſervations made on Fulham Church cannot be conſidered as very exact, becauſe Hanger Hill Tower itſelf was the object, inſtead of the Flagſtaff placed on it in 1787.

P. 250. in the 10. triangle, *for* 39963 *put* 48964.

In the 14. triangle, *for* 34413.6 *put* 34412.8.

P. 251. in the 20. triangle, *for* 39° 17′ 6″.5 *put* 39° 17′ 16″.5.

In the 27. triangle, there is a tranſpoſition of the two firſt angles, that oppoſite Fairlight ſhould ſtand oppoſite Goudhurſt.

P. 252. in the 28. triangle, *for* 5384 *put* 5585.

In the 35. triangle, *for* 23081.4 *put* 23018.4.

P. 254. in the 4. triangle, there is a tranſpoſition of the diſtances, 7198.2 ſhould ſtand oppoſite Hornſey Hill.

P. 255. in the 16. triangle, *for* 14390.2 and 6073.8 *put* 14390.8 and 6074.

P. 256. in the 22. triangle, *for* 8136.4 and 17641 *put* 8136 and 17640.3.

Neither of the angles of the 26. triangle was obſerved, becauſe St. Paul's is not ſeen from Greenwich Obſervatory. The diſtance of St. Paul's from Greenwich Obſervatory is alſo omitted. This diſtance is had from the VIII triangle, p. 173. and the 11. triangle, p. 250; from theſe we have,

		°	′	″	
the angle at	Norwood	42	15	26.5	hence Greenwich Ob. from St. Paul's 25655½ feet.
	Greenwich Ob.	82	41	1.1	
	St. Paul's	55	3	32.4	

The angles were determined thus:

	°	′	″	
Severndroog Caſtle	22	20	46	obſerved angles.
Greenwich Obſervatory	134	16	31	
Limehouſe Church	33	22	43	

Hence

Hence Greenwich Obſervatory from Limehouſe Church is 13999 feet.

Taking the ſum of 111° 56′ 50″ (in the VIII triangle, p. 173.) 82° 41′ 1″ and 134° 16′ 31″ from 360° there remains 31° 5′ 38″ for the angle at Greenwich Obſervatory between Limehouſe Church and St. Paul's; this, with the including ſides 13999 and 25655 (neglecting the fractions) will give 121° 1′ 42″ and 27° 52′ 40″, the other two angles.

P. 257. in the 31. triangle, *for* 6511 *put* 6925.4.

P. 259. The bearing of Greenwich from the meridian of St. Paul's, on which the other bearings in the Table depend, was found as follows:

	°	′	″
Angle at Greenwich Obſervatory between its meridian and Norwood .	38	7	16
between Norwood and St. Paul's	82	41	1.1
Hence the angle at Greenwich Obſervatory between the north meridian and St. Paul's .	59	11	42.9

This laſt angle, and its complement, with 25655½ feet the diſtance of Greenwich Obſervatory and St. Paul's, give 13138.5 for what St. Paul's is north from Greenwich, and 22036 for what it is weſt from the meridian.

On M. **Bouguer**'s ſpheroid 13138.5 feet anſwer to 2′ 9″.5 on the meridian; hence the latitude of the point on the meridian of Greenwich, where a great circle, paſſing through St. Paul's, falls perpendicular on that meridian, will be 51° 30′ 49″½: and taking 61251 fathoms for a degree perpendicular to the meridian (ſee Tab. p. 227.) we have (22036 feet) 3′ 35″.86 for the intercepted arc of that circle; this, and the complement of 51° 30′ 49″½ (as the legs of a ſpherical triangle) give 89° 55′ 29″ for the angle at St. Paul's, which, added to 30° 48′ 17″ (the complement of 59° 11′ 43″) gives 120° 43′ 46″, for the bearing of Greenwich Obſervatory, as in the Table.

P. 259. againſt St. Luke's, *for* 12° 57′ 53″.7 *put* 12° 37′ 37″.
Againſt Shoreditch Church, *for* 44° 57′ 39″.8 and 6746.4 *put* 44° 54′ 58″.8 and 6743.2.
Againſt Severndroog Caſtle, *for* 115° 28′ 50″.4 *put* 115° 25′ 50″.4.
Againſt Eltham Church, *for* 123° 46′ 4″.2 *put* 123° 46′ 4″.

P. 260. againſt Stretham Church, *for* 31793.5 *put* 31739.5.
Againſt Clapham Common, *for* 26° 29′ 56″.1 *put* 26°29′ 52″.
Againſt St. Bride's Church, *for* 1771.7 *put* 1687.6.
Againſt St. George's Bloomſbury, *for* 103° 15′ 20″.6 *put* 103° 15′ 50″.
Againſt the Tabernacle, *for* 107° 20′ 47″ *put* 107° 19′ 47″.
Againſt Highbury Houſe, *for* 178° 43′ 11″.6 *put* 178° 43′ 14″.6.

In Plate X. fig. 2. *p* is omitted at the concourſe of the meridians *eb*, *na*. Alſo a line from A to T in fig. 3. ib. *for* F *put* R.

Plate XI. Eltham Church is laid down too near the meridian of St. Paul's; and St. Bride's Church, Fleet-Street, ſhould ſtand on the north ſide of the weſt line. There are a few other corrections neceſſary on account of the errors in Tab. p. 259, 260.

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